On the complexity of scheduling checkpoints for computational workflows

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June 24, 2012

Motivation

Framework

- Application task graph, a DAG where nodes represent tasks and edges correspond to dependences between them.
- Application DAG to be executed on a failure-prone platform of p identical processors.

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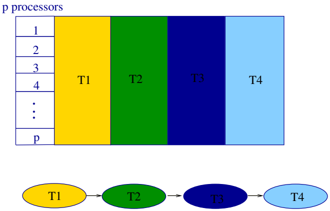
- Application DAG to be executed on a failure-prone platform of p identical processors.
- Each task is executed in parallel on the p processors.



- Resilience provided through coordinated checkpointing.

Objective and Questions

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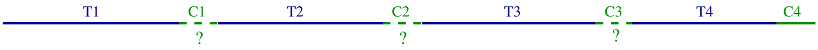
Questions :

- In which order should we execute the tasks?
- At the end of the execution of each task Ti, should we perform a checkpoint or should we proceed directly with the computation of another task?



State of the art

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- Maximizing the amount of work done before the first failure.
- NP-complete problem (in the weak sense) for uniform distributions.
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We solve the original problem that is minimizing the expected execution time (At least for Exponential failures)

Outline

1. Expected time needed to execute a work and to checkpoint it
2. Complexity of the general scheduling problem
3. Dynamic Programming algorithm for linear chains
4. Conclusion and extensions

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- D : Downtime (hardware replacement by spare, or software rejuvenation via rebooting)
- R: Recovery cost after failure
- A failure can happen during a checkpoint, a recovery, but not a downtime (otherwise replace D by 0 and R by $R + D$).

Problem statement

Compute the expected time $\mathbb{E}(T(\mathcal{W}, C, R))$ to execute a work of duration $\mathcal{W}$ followed by a checkpoint of duration C.

- Recursive Approach :

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$$\mathbb{E}(T(\mathcal{W}, C, R)) = e^{\lambda R}\left(\frac{1}{\lambda} + D\right)\left(e^{\lambda(\mathcal{W}+C)} - 1\right)$$

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The general scheduling problem is :

- Given a time bound K, can we find an ordering for the execution of several independent tasks, and decide after which tasks to checkpoint, so that the expected execution time does not exceed K?

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Proposition

Consider n independent tasks, T_1, ..., T_n, with task T_i of duration $\mathcal{W}_i$ for $1 \leq i \leq n$. All checkpoint and recovery times are equal to C, and there is no downtime ($D = 0$). The problem to schedule these tasks, and to decide after which tasks to checkpoint, so as to minimize the expected execution time, is NP-complete in the strong sense.

Proof of NP-completeness

We use a reduction from 3-PARTITION, which is NP-complete in the strong sense.

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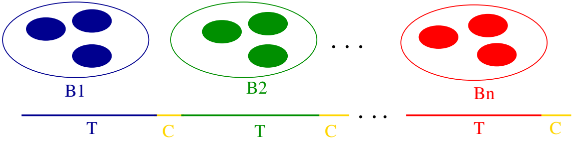
- General instance $\mathcal{I}_1$ of 3-PARTITION:
  given $3n$ integers $a_1, \ldots, a_{3n}$ and a number T such that $\sum_{1 \leq j \leq 3n} a_j = nT$, and $\frac{T}{4} < a_j < \frac{T}{2}$ for $1 \leq j \leq 3n$, does there exist a partition in n subsets $B_1, \ldots, B_n$ of $\{a_1, \ldots, a_{3n}\}$ such that for all $1 \leq i \leq n$, $\sum_{a_j \in B_i} a_j = T$. Note that necessarily in any solution, each B_i has cardinal 3.



Proof of NP-completeness

- Instance $\mathcal{I}_2$ of our problem :$3n$ independent tasks: $task_1$, ..., $task_{3n}$, $task_i$ being of size $\mathcal{W}_i = a_i$.We let:
  $\lambda = \frac{1}{2T}, \quad C = R = \frac{1}{\lambda}(\ln(2) - \frac{1}{2})$, and
  $D = 0, \quad K = n\frac{e^{\lambda C}}{\lambda}(e^{\lambda(T+C)} - 1)$.

$\mathcal{I}_1$ has a solution $\Longrightarrow$ $\mathcal{I}_2$ has a solution.

Suppose that $\mathcal{I}_1$ has a solution $B_1, \ldots, B_n$.
Propose the following solution:

- We execute the subsets $B_1, \ldots, B_n$ in any order;
- for each subset B_i, we schedule its three tasks in any order, and we checkpoint after the third one.

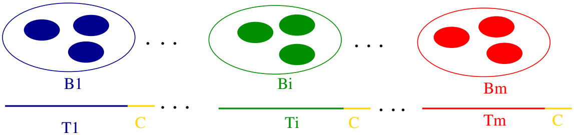
The expected total execution time is $\mathbb{E} = n\frac{e^{\lambda C}}{\lambda}(e^{\lambda(T+C)} - 1) = K$, hence a solution to $\mathcal{I}_2$.

Proof of NP-completeness

$\mathcal{I}_2$ has a solution $\Longrightarrow \mathcal{I}_1$ has a solution.

Suppose $\mathcal{I}_2$ has a solution:

- 3n independent tasks and a partition in m subsets $B_1, \ldots, B_m$
- $\sum_{i=1}^{m} T_i = nT$ and m checkpoints



The expected total execution time is:
$\mathbb{E} = \sum_{i=1}^{m} \frac{e^{\lambda C}}{\lambda}(e^{\lambda(T_i+C)} - 1)$, and $\mathbb{E} \leq K$.

We show that the minimum value of $\mathbb{E}$ is uniquely reached for $m = n$ and $T_i = T$ for all i, in which case $\mathbb{E} = K$.
So $B_1, \ldots, B_n$ is a solution for $\mathcal{I}_1$.

Outline

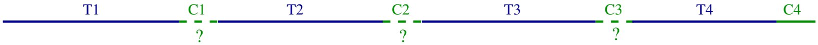
Problem statement

We want to compute the optimal expected execution time, that is:

- the expectation $\mathbb{E}$ of the time needed to process all the tasks of an applications whose DAG is a linear chain.

Problem:

- Decide whether to checkpoint or not after the completion of each given task.



Dynamic programming

T1 | ? C1 | T2 | ? C2 | T3 | ? C3 | T4 | C4

C1	C2	C3	C4
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

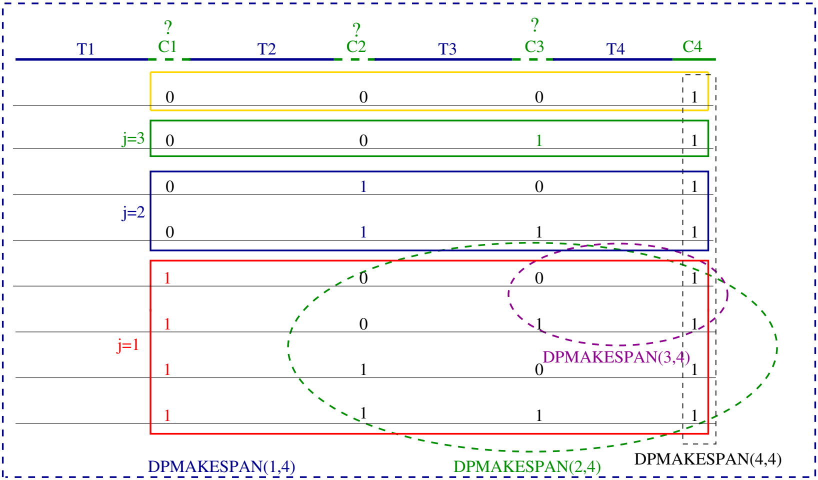
DPMAKESPAN(1,4)

Dynamic programming

Algorithm 1: *DPMAKESPAN*(x, n)

if $x = n$ **then**
  return $(\mathbb{E}(T(\mathcal{W}_n, C_n, R_{n-1})), n)$
$best \leftarrow \mathbb{E}(T(\sum_{i=x}^{n} \mathcal{W}_i, C_n, R_{x-1}))$
$numTask \leftarrow n$
for $j = x$ *to* $n - 1$ **do**
  $(exp\_succ, num\_Task) \leftarrow DPMAKESPAN(j + 1, n)$
  $Cur \leftarrow exp\_succ$
  $\quad + \mathbb{E}(T(\sum_{i=x}^{j} \mathcal{W}_i, C_j, R_{x-1}))$
  if $Cur < best$ **then**
    $best \leftarrow Cur$
    $numTask \leftarrow j$
return $(best, numTask)$

Dynamic programming



DPMAKESPAN(1,4)	DPMAKESPAN(2,4)	DPMAKESPAN(3,4)	DPMAKESPAN(4,4)

Linear complexity

Proposition

Algorithm1 provides the optimal solution for a linear chain of n tasks. Its complexity is $O(n^2)$.

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2. Complexity of the general scheduling problem
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Extensions

- General model of checkpointing costs:
  - The checkpoint after a task T_i may depend on T_i and on some other tasks that have been executed since the last checkpoint.
- Alleviating the *full parallelism* assumption:
  - Variable parallelism.
  - Ressource allocation problem.
- Using general failure laws than Exponential distributions:
  - First difficulty: Approximating the failure distribution of a platform of p processors.
  - Second difficulty: Estimating the expected execution time of a work W.

Conclusion

Important results:

- Closed-form formula for the expected execution time of a computational workflows followed by its checkpoint (using Exponential failure distribution).
- The strong NP-hardness of the problem for independent tasks and constant checkpoint costs.
- Dynamic programming algorithm for linear chains of tasks with arbitrary checkpoint costs.

Bibliography

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